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A Generalized Class of Jack-Knifed Estimator for Population Mean Using Two Auxiliary Variables under Measurement Errors

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Abstract: In this paper, we have proposed a generalized class of estimators of population mean, ratio and product of population means using auxiliary information of two variables in presence of measurement errors. Further, we also proposed the jack-knifedclass of unbiased estimators using Quenouille's method in respect of the above mentioned class of estimators. The bias and mean square error of the proposed classes are obtained. We also analyzed the properties of the generalized estimator in presence of measurement errors. Finally, some concluding remarks are made clearly demonstrating that some important class of estimators is special cases of the proposed study.

I. INTRODUCTION

Over the past several decades, statisticians are paying their attention towards the problem of estimation of parameters in the presence of measurement errors. In survey sampling, the properties of data usually presuppose that the observations are the correct measurements on characteristics being studied. However this assumption is not satisfied in many applications and data is contaminated with measurement errors, such as non-response errors, reporting errors and computing errors. These measurement errors make the result invalid, which are meant for no measurement error case. If measurement errors are very small and we can neglect it, then the statistical inference based on data observed continue to remain valid. On the contrary, when they are not appreciably small and negligible, the inferences may not be simply invalid and inaccurate but may often lead to unexpected, undesirable and unfortunate consequences (see Srivastava and Shalabh, 2001). Some important sources of measurement errors in survey data are discussed in Cochran (1968), Shalabh (1997), and Singh and Karpe (2008, 2010) studied some estimators of population mean under measurement errors.

For a simple random sample of size *n*, let $\left(x_{1j}, x_{2j}, y_j\right)$ be the pair of values instead of the true values $\left(X_{1j}, X_{2j}, Y_j\right)$ on the characteristics $\left(X_1, X_2, Y\right)$ respectively.Let the observational or measurement error be defined as

$u_j = y_j - Y_j \; j=1,2, \ldots, n$ (1.1)

$v_{1j} = x_{1j} - X_{1j}$

$v_{2j} = x_{2j} - X_{2j}$ (1.2)

The generalized estimator of population mean μ_Y, using mean μ_1 and μ_2 of auxiliary variable X_1 and X_2 respectively, is given by

$$\bar{y}_a = f\left(u, v, w\right) \quad (1.3)$$

where $u = \bar{y}$, $v = \frac{\bar{x}_1}{\mu_1}$ and $w = \frac{\bar{x}_2}{\mu_2}$

(i) $f\left(\bar{Y}, 1, 1\right) = \mu_Y$

(ii) The function is continuous and bounded in the closed interval *R* of real line.

(iii) The first and second order partial derivatives of the function are exist and are continuous and bounded in *R*

where $u = \bar{y}, v = \frac{\bar{x}_1}{\mu_1}$ and $w = \frac{\bar{x}_2}{\mu_2}$ and $\bar{x}_1, \bar{x}_2$ and $\bar{y}$ are the sample mean of X_1, X_2 and Y_j respectively for a simple random variable of size *n*.

$$\bar{y} = \frac{1}{n}\left[\sum_{j=1}^{n} Y_j\right]$$

$$= \frac{1}{n}\left[\sum_{j=1}^{n} u_j + Y_j - \mu_Y + \mu_Y\right]$$

$$= \frac{1}{\sqrt{n}}\left[W_u + W_Y\right] + \mu_Y \quad (1.4)$$

Similarly,

$$\bar{x}_1 = \frac{1}{\sqrt{n}}\left[W_{v_1} + W_{x_1}\right] + \mu_1 \quad (1.5)$$

and

$$\bar{x}_2 = \frac{1}{\sqrt{n}}\left[W_{v_2} + W_{x_2}\right] + \mu_2 \quad (1.6)$$

Then, putting these values in the generalized estimator (1.3), we have

$$\bar{y}_a = f(u,v,w)$$

$$= f\left[\bar{y},\left(\frac{\bar{x}_1}{\mu_1}\right),\left(\frac{\bar{x}_2}{\mu_2}\right)\right]$$

$$= \left[f\left(\bar{Y},\bar{X},\bar{X}\right) + \left(\bar{y} - \mu_Y\right)\frac{\partial f}{\partial u} + \left(\frac{\bar{x}_1}{\mu_1} - 1\right)\frac{\partial f}{\partial v} + \left(\frac{\bar{x}_2}{\mu_2} - 1\right)\frac{\partial f}{\partial w} \right.$$

$$+ \frac{1}{2!}\left\{\left(\bar{y} - \mu_Y\right)^2 \frac{\partial^2 f}{\partial u^2} + \left(\frac{\bar{x}_1}{\mu_1} - 1\right)^2 \frac{\partial^2 f}{\partial v^2} + \left(\frac{\bar{x}_2}{\mu_2} - 1\right)^2 \frac{\partial^2 f}{\partial w^2} + 2\left(\bar{y} - \mu_Y\right)\left(\frac{\bar{x}_1}{\mu_1} - 1\right)\frac{\partial^2 f}{\partial u \partial v}\right.$$

$$\left.\left. + 2\left(\frac{\bar{x}_1}{\mu_1} - 1\right)\left(\frac{\bar{x}_2}{\mu_2} - 1\right)\frac{\partial^2 f}{\partial v \partial w} + 2\left(\frac{\bar{x}_2}{\mu_2} - 1\right)\left(\bar{y} - \mu_Y\right)\frac{\partial^2 f}{\partial w \partial u}\right\} + \dots\right]$$

$$= \left[\mu_Y + \frac{\left(W_u + W_Y\right)}{\sqrt{n}} f_1 + \frac{\left(W_{v_1} + W_{x_1}\right)}{\mu_1\sqrt{n}} f_2 + \frac{\left(W_{v_2} + W_{x_2}\right)}{\mu_2\sqrt{n}} f_3 + \frac{1}{2!}\left\{\frac{\left(W_u + W_Y\right)^2}{n} f_{200}\right.\right.$$

$$+ \frac{\left(W_{v_1} + W_{x_1}\right)^2}{\mu_1^2 n} f_{020} + \frac{\left(W_{v_2} + W_{x_2}\right)^2}{\mu_2^2 n} f_{002} + 2\frac{\left(W_u + W_Y\right)\left(W_{v_1} + W_{x_1}\right)}{n\mu_1} f_{110}$$

$$\left.\left. + \frac{\left(W_{v_1} + W_{x_1}\right)\left(W_{v_2} + W_{x_2}\right)}{\mu_1\mu_2} f_{011} + 2\frac{\left(W_u + W_Y\right)\left(W_{v_2} + W_{x_2}\right)}{\mu_2} f_{101}\right\} + \dots\right]$$

$$\left(\bar{y}_a - \mu_Y\right) = \left[\frac{\left(W_u + W_y\right)}{\sqrt{n}} f_1 + \frac{\left(W_{v_1} + W_{x_1}\right)}{\mu_1\sqrt{n}} f_2 + \frac{\left(W_{v_2} + W_{x_2}\right)}{\mu_2\sqrt{n}} f_3 + \frac{1}{2!}\left\{\frac{\left(W_{v_1} + W_{x_1}\right)^2}{\mu_1^2 n} f_{020}\right.\right.$$

$$+ \frac{\left(W_{v_2} + W_{x_2}\right)^2}{\mu_2^2 n} f_{002} + 2\frac{\left(W_u + W_y\right)\left(W_{v_1} + W_{x_1}\right)}{n\mu_1} f_{110}$$

$$\left.\left. + \frac{\left(W_{v_1} + W_{x_1}\right)\left(W_{v_2} + W_{x_2}\right)}{\mu_1\mu_2} f_{011} + 2\frac{\left(W_u + W_y\right)\left(W_{v_2} + W_{x_2}\right)}{\mu_2} f_{101}\right\}\right] \quad (1.7)$$

II. BIAS AND MEAN SQUARE ERROR OF THE PROPOSED CLASS OF GENERALIZED ESTIMATOR

Taking expectation on both side of (1.7), we have

$$E\left(\bar{y}_a - \mu_Y\right) = E\left[\frac{\left(W_u + W_y\right)}{\sqrt{n}} f_1 + \frac{\left(W_{v_1} + W_{x_1}\right)}{\mu_1\sqrt{n}} f_2 + \frac{\left(W_{v_2} + W_{x_2}\right)}{\mu_2\sqrt{n}} f_3 + \frac{1}{2!}\left\{\frac{\left(W_u + W_y\right)^2}{n} f_{200}\right.\right.$$

$$+ \frac{\left(W_{v_1} + W_{x_1}\right)^2}{\mu_1^2 n} f_{020} + \frac{\left(W_{v_2} + W_{x_2}\right)^2}{\mu_2^2 n} f_{002} + 2\frac{\left(W_u + W_y\right)\left(W_{v_1} + W_{x_1}\right)}{n\mu_1} f_{110}$$

$$\left.\left. + \frac{\left(W_{v_1} + W_{x_1}\right)\left(W_{v_2} + W_{x_2}\right)}{\mu_1\mu_2} f_{011} + 2\frac{\left(W_u + W_y\right)\left(W_{v_2} + W_{x_2}\right)}{\mu_2} f_{101}\right\} + \ldots\right]$$

$$= \frac{1}{2n}\left[\frac{\left(\sigma_{v_1}^2 + \sigma_{x_1}^2\right)}{\mu_1^2} f_{020} + \frac{\left(\sigma_{v_2}^2 + \sigma_{x_2}^2\right)}{\mu_2^2} f_{002}\right]$$

$$+\left[\frac{\sigma_{(y,x_1)}}{\mu_1} f_{100} + \frac{\sigma_{(x_1,x_2)} f_{011}}{\mu_1\mu_2} + \frac{\sigma_{(y,x_2)}}{\mu_2} f_{101}\right]$$

Hence, bias of $\bar{y}_a$ is given by

$$Bias\left(\bar{y}_a\right) = \frac{1}{2n}\left[\frac{1}{\mu_1^2}\sigma_{x_1}^2 f_{020} + \frac{1}{\mu_2^2}\sigma_{x_2}^2 f_{002} + 2\left\{\frac{\sigma_{(y,x_1)}}{\mu_1} f_{100} + \frac{\sigma_{(x_1,x_2)} f_{011}}{\mu_1\mu_2} + \frac{\sigma_{(y,x_2)}}{\mu_2} f_{101}\right\}\right]$$

$$+\frac{1}{2n}\left[\frac{\sigma_{v_1}^2}{\mu_1^2} f_{020} + \frac{\sigma_{v_2}^2}{\mu_2^2} f_{002}\right] \quad (2.1)$$

Now, squaring (1.7) on both sides and then taking expectation, we have

$$MSE\left(\bar{y}_a\right) = E\left(\bar{y}_a - \mu_Y\right)^2$$

$$= E\left[\frac{\left(W_u + W_y\right)}{\sqrt{n}} f_1 + \frac{\left(W_{v_1} + W_{x_1}\right)}{\mu_1\sqrt{n}} f_2 + \frac{\left(W_{v_2} + W_{x_2}\right)}{\mu_2\sqrt{n}} f_3\right]^2$$

$$= E\left[\frac{\left(W_u + W_y\right)^2}{n}\left(f_1\right)^2 + \frac{\left(W_{v_1} + W_{x_1}\right)^2}{\mu_1^2 n}\left(f_2\right)^2 + \frac{\left(W_{v_2} + W_{x_2}\right)^2}{\mu_2^2 n}\left(f_3\right)^2\right.$$

$$+2\left\{\frac{\left(W_u + W_y\right)}{\sqrt{n}}\frac{\left(W_{v_1} + W_{x_1}\right)}{\mu_1\sqrt{n}} f_1 f_2 + \frac{\left(W_{v_1} + W_{x_1}\right)}{\mu_1\sqrt{n}}\frac{\left(W_{v_2} + W_{x_2}\right)}{\mu_2\sqrt{n}} f_2 f_3\right.$$

$$\left.\left. + \frac{\left(W_{v_2} + W_{x_2}\right)}{\mu_2\sqrt{n}}\frac{\left(W_u + W_y\right)}{\sqrt{n}} f_3 f_1\right\}\right]$$

$$= \left[\frac{\left(\sigma_u^2 + \sigma_y^2\right)}{n}\left(f_1\right)^2 + \frac{\left(\sigma_{v_1}^2 + \sigma_{x_1}^2\right)}{n\mu_1^2}\left(f_2\right)^2 + \frac{\left(\sigma_{v_2}^2 + \sigma_{x_2}^2\right)}{n\mu_2^2}\left(f_3\right)^2\right]$$

$$+\frac{2}{n}\left[\frac{\sigma_{(y,x_1)}}{\mu_1} f_1 f_2 + \frac{\sigma_{(x_1,x_2)}}{\mu_1\mu_2} f_2 f_3 + \frac{\sigma_{(y,x_2)}}{\mu_2} f_3 f_1\right]$$

$$= \frac{1}{n}\left[\sigma_y^2\left(f_1\right)^2+\frac{\sigma_{x_1}^2}{\mu_1^2}\left(f_2\right)^2+\frac{\sigma_{x_2}^2}{\mu_2^2}\left(f_3\right)^2+2\left\{\frac{\sigma_{(y,x_1)}}{\mu_1}f_1f_2+\frac{\sigma_{(x_1,x_2)}}{\mu_1\mu_2}f_2f_3+\frac{\sigma_{(y,x_2)}}{\mu_2}f_3f_1\right\}\right]$$
$$+\frac{1}{n}\left[\sigma_u^2\left(f_1\right)^2+\frac{\sigma_{v_1}^2}{\mu_1^2}\left(f_2\right)^2+\frac{\sigma_{v_2}^2}{\mu_2^2}\left(f_3\right)^2\right] \quad (2.2)$$

III. PROPOSED GENERALIZED CLASS OF UNBIASED JACK-KNIFE ESTIMATORS

Now, we consider a simple random sample of size *2m* and split this sample randomly into two sub samples each of size *m*. Let $\left(\overline{x}_{12m},\overline{x}_{22m},\overline{y}_{2m}\right)$ be the sample means of values on $\left(\mathrm{Y},\mathrm{X}\right)$ respectively for the entire sample of size *2m* and $\left(\overline{x}_{1m}^{(\mathrm{i})},\overline{x}_{2m}^{(\mathrm{i})},\overline{y}_{m}^{(\mathrm{i})}\right)$ be the sample means values of $\left(X_1,X_2,Y\right)$ respectively for $i^{th}\left(i=1,2\right)$ sub-sample of size *m.*

The generalized estimator of population mean μ_Y using mean μ_1 and μ_2 of auxiliary variable X_1 and X_2 respectively is

$$\overline{y}_a=f\left(u,v,w\right)=f\left[\overline{y},\frac{\overline{x}_{1m}}{\mu_1},\frac{\overline{x}_{2m}}{\mu_2}\right] \quad (3.1)$$

Also, $\left(\overline{x}_{1m},\overline{x}_{2m},\overline{y}_{m}\right)$ be the sample means values of $\left(X_1,X_2,Y\right)$ respectively for sample of size *m* so that $u=\overline{y},\ v=\left(\frac{\overline{x}_{1m}}{\mu_1}\right)$ and $w=\left(\frac{\overline{x}_{2m}}{\mu_2}\right)$ and $f\left(u_m,v_m,w_m\right)$ satisfying the validity conditions of Taylor's series expansion is a bounded function of *u, v* and *w* such that $f\left(\mu_Y,1,1\right)=\mu_Y$.

Let $\overline{y}_a^{(3)}$ is a generalized estimator for the entire sample of size *2m*; $\overline{y}_a^{(1)}$ and $\overline{y}_a^{(2)}$ be the generalized estimators for the two randomly split sub-samples of size *m*each.Thus,

$$\overline{y}_a^{(3)}=f\left[\overline{y}_{2m},\left(\frac{\overline{x}_{1,2m}}{\mu_1}\right),\left(\frac{\overline{x}_{2,2m}}{\mu_2}\right)\right]$$
$$=f\left(z_3,z_3',z_3''\right) \quad (3.2)$$
$$\overline{y}_a^{(1)}=f\left[\overline{y}_m^{(1)},\left(\frac{\overline{x}_{1,m}^{(1)}}{\mu_1}\right),\left(\frac{\overline{x}_{2,m}^{(1)}}{\mu_2}\right)\right]$$
$$=f\left(z_1,z_1',z_1''\right) \quad (3.3)$$
$$\overline{y}_a^{(2)}=f\left[\overline{y}_m^{(1)},\left(\frac{\overline{x}_{1,m}^{(2)}}{\mu_1}\right),\left(\frac{\overline{x}_{2,m}^{(2)}}{\mu_2}\right)\right]$$
$$=f\left(z_2,z_2',z_2''\right) \quad (3.4)$$

where

$z_3=\overline{y}_{2m},z_3'=\left(\frac{\overline{x}_{1,2m}}{\mu_1}\right),\ z_3''=\left(\frac{\overline{x}_{2,2m}}{\mu_2}\right),\ z_1=\overline{y}_m^{(1)},z_1'=\left(\frac{\overline{x}_{1,m}^{(1)}}{\mu x_1}\right),z_1''=\left(\frac{\overline{x}_{2,m}^{(1)}}{\mu x_2}\right),$

$z_2=\overline{y}_m^{(1)},z_2'=\left(\frac{\overline{x}_{1,m}^{(2)}}{\mu_1}\right),z_2''=\left(\frac{\overline{x}_{2,m}^{(2)}}{\mu_2}\right)$, $f\left(z_3,z_3',z_3''\right)$, $f\left(z_1,z_1',z_1''\right)$ and $f\left(z_2,z_2',z_2''\right)$ are the bounded functions of $\left(z_3,z_3',z_3''\right),\left(z_1,z_1',z_1''\right)$ and $\left(z_2,z_2',z_2''\right)$ respectively satisfying the regularity of conditions of

$f\left(u,v,w\right)$ in (1.3) involved in generalized estimator $\overline{y}_a$ .We can easily define the following terms similarly as mentioned in (1.4), (1.5) and (1.6), so that

$$\overline{y}_{2m}=\frac{1}{2n}\left[\sum_{j=1}^{2m}Y_j\right]$$

$$=\frac{1}{\left(2m\right)^{1/2}}\left[\frac{1}{\left(2m\right)^{1/2}}\sum_{j=1}^{2m}y_j-Y_j+Y_j-\mu_Y+\mu_Y\right]$$

$$=\frac{1}{\left(2m\right)^{1/2}}\left[\frac{1}{\left(2m\right)^{1/2}}\sum_{j=1}^{2m}u_j+(Y_j-\mu_Y)+\mu_Y\right]$$

$$=\mu_Y+\frac{1}{\left(2m\right)^{1/2}}\left(W_u+W_y\right) \quad (3.5)$$

where $W_u=\frac{1}{\left(2m\right)^{1/2}}\sum_{j=1}^{2m}u_j$ and $W_Y=\frac{1}{\left(2m\right)^{1/2}}\sum_{j=1}^{2m}(Y_j-\mu_Y)$ are of order $O_p(1)$.Similarly, we also define $W_{v_1},W_{x_1},W_{V_2},W_{X_2},W_u^{(1)},W_y^{(1)},W_u^{(2)},W_y^{(2)},W_{v_1}^{(1)},\ W_{x_1}^{(1)},W_{v_1}^{(2)},W_{x_1}^{(2)},W_{v_2}^{(1)},W_{x_2}^{(1)},W_{v_2}^{(2)}\ and\ W_{x_2}^{(2)}$ as for w_u and w_Y .

$$\overline{x}_{1,2m}=\frac{1}{\sqrt{2m}}\left[W_{v_1}+W_{x_1}\right]+\mu_1 \quad (3.6)$$

$$\overline{x}_{2,2m}=\frac{1}{\sqrt{2m}}\left[W_{V_2}+W_{X_2}\right]+\mu_2 \quad (3.7)$$

$$\overline{y}_m^{(1)}=\frac{1}{\sqrt{m}}\left[W_u^{(1)}+W_y^{(1)}\right]+\mu_Y \quad (3.8)$$

$$\overline{x}_{1,m}^{(1)}=\frac{1}{\sqrt{m}}\left[W_{v_1}^{(1)}+W_{x_1}^{(1)}\right]+\mu_1 \quad (3.9)$$

$$\overline{x}_{2,m}^{(1)}=\frac{1}{\sqrt{m}}\left[W_{v_2}^{(1)}+W_{x_2}^{(1)}\right]+\mu_2 \quad (3.10)$$

$$\overline{y}_m^{(2)}=\frac{1}{\sqrt{m}}\left[W_u^{(2)}+W_y^{(2)}\right]+\mu_Y \quad (3.11)$$

$$\overline{x}_{1,m}^{(2)}=\frac{1}{\sqrt{m}}\left[W_{v_1}^{(2)}+W_{x_1}^{(2)}\right]+\mu_1 \quad (3.12)$$

$$\overline{x}_{2,m}^{(2)}=\frac{1}{\sqrt{m}}\left[W_{v_2}^{(2)}+W_{x_2}^{(2)}\right]+\mu_2 \quad (3.13)$$

On expanding $f\left(z_3,z_3',z_3''\right)$, $f\left(z_1,z_1',z_1''\right)$ and $f\left(z_2,z_2',z_2''\right)$ in third order taylor's series about the point $z_i=\mu_Y.z_i'=1.z_i''=1$ and noting that $f\left(\mu_Y,1,1\right)=\mu_Y$ for i=1, 2, 3, we have

$$\overline{y}_a^{(3)}=\Bigg[\ f\left(\mu_Y,1,1\right)+\left(z_3-1\right)\frac{\partial f}{\partial z_3}+\left(z_3'-1\right)\frac{\partial f}{\partial z_3'}+\left(z_3''-1\right)\frac{\partial f}{\partial z''}+\frac{1}{2!}\Bigg\{\left(z_3-1\right)^2\frac{\partial^2 f}{\partial z_3^2}$$

$$+\left(z_3'-1\right)^2\frac{\partial^2 f}{\partial z_3'^2}+\left(z_3''-1\right)^2\frac{\partial^2 f}{\partial z''^2}+2\left(z_3-1\right)\left(z_3'-1\right)\frac{\partial^2 f}{\partial z_3\partial z_3'}$$

$$+2\left(z_3'-1\right)\left(z_3''-1\right)\frac{\partial^2 f}{\partial z_3'\partial z''}+\left(z_3''-1\right)\left(z_3-1\right)\frac{\partial^2 f}{\partial z''\partial z_3}\Bigg\}+...\Bigg]$$

$$\bar{y}_a^{(3)}=\Bigg[\mu_Y+\frac{\left(w_u+w_y\right)}{\left(2m\right)^{1/2}}f_1+\frac{\left(W_{v_1}+W_{x_1}\right)}{\mu_1\left(2m\right)^{1/2}}f_2+\frac{\left(W_{v_2}+W_{x_2}\right)}{\mu_2\left(2m\right)^{1/2}}f_3+\frac{1}{2!2m}\Bigg\{\left(w_u+w_y\right)^2 f_{200}$$

$$+\frac{\left(W_{v_1}+W_{x_1}\right)^2}{\mu_1^2}f_{020}+\frac{\left(w_{v_2}+w_{x_2}\right)^2}{\mu_2^2}f_{002}\Bigg\}+2\Bigg\{\frac{\left(w_u+w_y\right)\left(W_{v_1}+W_{x_1}\right)}{\mu_1}f_{110}$$

$$+\frac{\left(W_{v_1}+W_{x_1}\right)\left(W_{v_2}+W_{x_2}\right)}{\mu_1\mu_2}f_{011}+\frac{\left(W_{v_2}+W_{x_2}\right)\left(w_u+w_y\right)}{\mu_2}f_{101}\Bigg\}+...\Bigg] \quad (3.14)$$

Similarly, we have

$$\bar{y}_a^{(1)}=\Bigg[\mu_Y+\frac{\left(w_{u^{(1)}}+w_{y^{(1)}}\right)}{\left(m\right)^{1/2}}f_1+\frac{\left(W_{v_1^{(1)}}+W_{x_1^{(1)}}\right)}{\mu_1\left(m\right)^{1/2}}f_2+\frac{\left(W_{v_2^{(1)}}+W_{x_2^{(1)}}\right)}{\mu_2\left(m\right)^{1/2}}f_3+\frac{1}{2!m}\Bigg\{\left(w_{u^{(1)}}+w_{Y^{(1)}}\right)^2 f_{200}$$

$$+\frac{\left(W_{v_1^{(1)}}+W_{x_1^{(1)}}\right)^2}{\mu_1^2}f_{020}+\frac{\left(w_{v_2^{(1)}}+w_{x_2^{(1)}}\right)^2}{\mu_2^2}f_{002}\Bigg\}+2\Bigg\{\frac{\left(w_{u^{(1)}}+w_{Y^{(1)}}\right)\left(W_{v_1^{(1)}}+W_{x_1^{(1)}}\right)}{\mu_1}f_{110}$$

$$+\frac{\left(W_{v_1^{(1)}}+W_{x_1^{(1)}}\right)\left(W_{v_2^{(1)}}+W_{x_2^{(1)}}\right)}{\mu_1\mu_2}f_{011}+\frac{\left(W_{v_2^{(1)}}+W_{x_2^{(1)}}\right)\left(w_{u^{(1)}}+w_{y^{(1)}}\right)}{\mu_2}f_{101}\Bigg\}+...\Bigg] \quad (3.15)$$

$$\bar{y}_a^{(2)}=\Bigg[\mu_Y+\frac{\left(w_{u^{(2)}}+w_{y^{(2)}}\right)}{\left(m\right)^{1/2}}f_1+\frac{\left(W_{v_1^{(2)}}+W_{x_1^{(2)}}\right)}{\mu_1\left(m\right)^{1/2}}f_2+\frac{\left(W_{v_2^{(2)}}+W_{x_2^{(2)}}\right)}{\mu_2\left(m\right)^{1/2}}f_3+\frac{1}{2!m}\Bigg\{\left(w_{u^{(2)}}+w_{y^{(2)}}\right)^2 f_{200}$$

$$+\frac{\left(W_{v_1^{(2)}}+W_{x_1^{(2)}}\right)^2}{\mu_1^2}f_{020}+\frac{\left(w_{v_2^{(2)}}+w_{x_2^{(2)}}\right)^2}{\mu_2^2}f_{002}\Bigg\}+2\Bigg\{\frac{\left(w_{u^{(2)}}+w_{y^{(2)}}\right)\left(W_{v_1^{(2)}}+W_{x_1^{(2)}}\right)}{\mu_1}f_{110}$$

$$+\frac{\left(W_{v_1^{(2)}}+W_{x_1^{(2)}}\right)\left(W_{v_2^{(2)}}+W_{x_2^{(2)}}\right)}{\mu_1\mu_2}f_{011}+\frac{\left(W_{v_2^{(2)}}+W_{x_2^{(2)}}\right)\left(w_{u^{(2)}}+w_{y^{(2)}}\right)}{\mu_2}f_{101}\Bigg\}+...\Bigg] \quad (3.16)$$

As defined in Sukhatme and Sukhatme (Chapter IV, page 162), for *N*large, we propose the Jack-knifed generalized estimator $\bar{y}_{aj}$ under measurement error given by

$$\bar{y}_{aj}=2\bar{y}_a^{(3)}-\frac{1}{2}\left(\bar{y}_a^{(1)}+\bar{y}_a^{(2)}\right) \quad (3.17)$$

$$\bar{y}_{aj}=2\Bigg[\mu_Y+\frac{\left(w_u+w_y\right)}{\left(2m\right)^{1/2}}f_1+\frac{\left(W_{v_1}+W_{x_1}\right)}{\mu_1\left(2m\right)^{1/2}}f_2+\frac{\left(W_{v_2}+W_{x_2}\right)}{\mu_2\left(2m\right)^{1/2}}f_3+\frac{1}{2!2m}\Bigg\{\left(w_u+w_y\right)^2 f_{200}$$

$$+\frac{\left(W_{v_1}+W_{x_1}\right)^2}{\mu_1^2}f_{020}+\frac{\left(w_{v_2}+w_{x_2}\right)^2}{\mu_2^2}f_{002}\Bigg\}+2\Bigg\{\frac{\left(w_u+w_y\right)\left(W_{v_1}+W_{x_1}\right)}{\mu_1}f_{110}$$

$$+\frac{\left(W_{v_1}+W_{x_1}\right)\left(W_{v_2}+W_{x_2}\right)}{\mu_1\mu_2}f_{011}+\frac{\left(W_{v_2}+W_{x_2}\right)\left(w_u+w_y\right)}{\mu_2}f_{101}\Bigg\}+\ldots\Bigg]$$

$$-\frac{1}{2}\Bigg[\mu_Y+\frac{\left(w_{u^{(1)}}+w_{y^{(1)}}\right)}{(m)^{1/2}}f_1+\frac{\left(W_{v_1^{(1)}}+W_{x_1^{(1)}}\right)}{\mu_1(m)^{1/2}}f_2+\frac{\left(W_{v_2^{(1)}}+W_{x_2^{(1)}}\right)}{\mu_2(m)^{1/2}}f_3+\frac{1}{2!m}\Bigg\{\left(w_{u^{(1)}}+w_{y^{(1)}}\right)^2f_{200}$$

$$+\frac{\left(W_{v_1^{(1)}}+W_{x_1^{(1)}}\right)^2}{\mu_1^2}f_{020}+\frac{\left(w_{v_2^{(1)}}+w_{x_2^{(1)}}\right)^2}{\mu_2^2}f_{002}\Bigg\}+2\Bigg\{\frac{\left(w_{u^{(1)}}+w_{Y^{(1)}}\right)\left(W_{v_1^{(1)}}+W_{x_1^{(1)}}\right)}{\mu_1}f_{110}$$

$$+\frac{\left(W_{v_1^{(1)}}+W_{x_1^{(1)}}\right)\left(W_{v_2^{(1)}}+W_{x_2^{(1)}}\right)}{\mu_1\mu_2}f_{011}+\frac{\left(W_{v_2^{(1)}}+W_{x_2^{(1)}}\right)\left(w_{u^{(1)}}+w_{y^{(1)}}\right)}{\mu_2}f_{101}\Bigg\}+\ldots$$

$$+\mu_Y+\frac{\left(w_{u^{(2)}}+w_{y^{(2)}}\right)}{(m)^{1/2}}f_1+\frac{\left(W_{v_1^{(2)}}+W_{x_1^{(2)}}\right)}{\mu_1(m)^{1/2}}f_2+\frac{\left(W_{v_2^{(2)}}+W_{x_2^{(2)}}\right)}{\mu_2(m)^{1/2}}f_3+\frac{1}{2!m}\Bigg\{\left(w_{u^{(2)}}+w_{y^{(2)}}\right)^2f_{200}$$

$$+\frac{\left(W_{v_1^{(2)}}+W_{x_1^{(2)}}\right)^2}{\mu_1^2}f_{020}+\frac{\left(w_{v_2^{(2)}}+w_{x_2^{(2)}}\right)^2}{\mu_2^2}f_{002}\Bigg\}+2\Bigg\{\frac{\left(w_{u^{(2)}}+w_{y^{(2)}}\right)\left(W_{v_1^{(2)}}+W_{x_1^{(2)}}\right)}{\mu_1}f_{110}$$

$$+\frac{\left(W_{v_1^{(2)}}+W_{x_1^{(2)}}\right)\left(W_{v_2^{(2)}}+W_{x_2^{(2)}}\right)}{\mu_1\mu_2}f_{011}+\frac{\left(W_{v_2^{(2)}}+W_{x_2^{(2)}}\right)\left(w_{u^{(2)}}+w_{y^{(2)}}\right)}{\mu_2}f_{101}\Bigg\}+\ldots\Bigg]$$

$$\overline{y}_{aj}-\mu_Y=\frac{2}{(2m)^{1/2}}\left[\left(W_u+W_y\right)f_1+\frac{1}{\mu_1}\left(W_{v_1}+W_{x_1}\right)f_2+\frac{1}{\mu_2}\left(W_{v_2}+W_{x_2}\right)f_3\right]+\frac{2}{2!2m}\Bigg[\left(W_u+W_y\right)^2f_{200}$$

$$+\frac{1}{\mu_1^2}\left(W_{v_1}+W_{x_1}\right)^2f_{020}+\frac{1}{\mu_2^2}\left(W_{v_2}+W_{x_2}\right)^2f_{002}+2\Bigg\{\frac{1}{\mu_1}\left(W_u+W_y\right)\left(W_{v_1}+W_{x_1}\right)f_{110}$$

$$+\frac{1}{\mu_1\mu_2}\left(W_{v_1}+W_{x_1}\right)\left(W_{v_2}+W_{x_2}\right)f_{011}+\frac{1}{\mu_2}\left(W_{v_2}+W_{x_2}\right)\left(W_u+W_Y\right)f_{101}\Bigg]$$

$$-\frac{1}{2}\Bigg[\frac{1}{(m)^{1/2}}\Bigg\{\left(w_{u^{(1)}}+w_{y^{(1)}}\right)f_1+\frac{1}{\mu_1}\left(W_{v_1^{(1)}}+W_{x_1^{(1)}}\right)f_2+\frac{1}{\mu_2}\left(W_{v_2^{(1)}}+W_{x_2^{(1)}}\right)f_3\Bigg\}$$

$$+\frac{1}{2!m}\Bigg\{\left(w_{u^{(1)}}+w_{y^{(1)}}\right)^2f_{200}+\frac{1}{\mu_1^2}\left(W_{v_1^{(1)}}+W_{x_1^{(1)}}\right)^2f_{020}+\frac{1}{\mu_2^2}\left(W_{v_2^{(1)}}+W_{x_2^{(1)}}\right)^2f_{002}\Bigg\}$$

$$+2\left\{\frac{1}{\mu_1}\left(w_{u^{(1)}}+w_{y^{(1)}}\right)\left(W_{v_1^{(1)}}+W_{x_1^{(1)}}\right)f_{110}+\frac{1}{\mu_1\mu_2}\left(W_{v_1^{(1)}}+W_{x_1^{(1)}}\right)\left(W_{v_2^{(1)}}+W_{x_2^{(1)}}\right)f_{011}\right.$$

$$\left.+\frac{1}{\mu_2}\left(W_{v_2^{(1)}}+W_{x_2^{(1)}}\right)\left(w_{u^{(1)}}+w_{y^{(1)}}\right)f_{101}\right\}$$

$$+\frac{1}{(m)^{1/2}}\left\{\left(w_{u^{(2)}}+w_{y^{(2)}}\right)f_1+\frac{1}{\mu_1}\left(W_{v_1^{(2)}}+W_{x_1^{(2)}}\right)f_2+\frac{1}{\mu_2}\left(W_{v_2^{(2)}}+W_{x_2^{(2)}}\right)f_3\right\}$$

$$+\frac{1}{2!m}\left\{\left(w_{u^{(2)}}+w_{y^{(2)}}\right)^2 f_{200}+\frac{1}{\mu_1^2}\left(W_{v_1^{(2)}}+W_{x_1^{(2)}}\right)^2 f_{020}+\frac{1}{\mu_2^2}\left(W_{v_2^{(2)}}+W_{x_2^{(2)}}\right)^2 f_{002}\right\}$$

$$+2\left\{\frac{1}{\mu_1}\left(w_{u^{(2)}}+w_{y^{(2)}}\right)\left(W_{v_1^{(2)}}+W_{x_1^{(2)}}\right)f_{110}+\frac{1}{\mu_1\mu_2}\left(W_{v_1^{(2)}}+W_{x_1^{(2)}}\right)\left(W_{v_2^{(2)}}+W_{x_2^{(2)}}\right)f_{011}\right.$$

$$\left.\left.+\frac{1}{\mu_2}\left(W_{v_2^{(2)}}+W_{x_2^{(2)}}\right)\left(w_{u^{(2)}}+w_{y^{(2)}}\right)f_{101}\right\}\right] \qquad (3.18)$$

Taking expectation on both sides of (3.18), we have

$$E\left(\bar{y}_{aj}-\mu_Y\right)=\left[\frac{2}{2!2m}\left(\sigma_u^2+\sigma_y^2\right)f_{200}+\frac{1}{\mu_1^2}\left(\sigma_{v_1}^2+\sigma_{x_1}^2\right)f_{020}+\frac{1}{\mu_2^2}\left(\sigma_{v_2}^2+\sigma_{x_2}^2\right)f_{002}\right.$$

$$\left.+\frac{2}{\mu_1}\sigma_{(y,x_1)}f_{110}+\frac{2}{\mu_2}\sigma_{(y,x_2)}f_{101}+\frac{2}{\mu_1\mu_2}\sigma_{(x_1,x_2)}f_{011}\right]$$

$$-\frac{1}{2}\left[\frac{2}{2!m}\left(\sigma_u^2+\sigma_y^2\right)f_{200}+\frac{1}{\mu_1^2}\left(\sigma_{v_1}^2+\sigma_{x_1}^2\right)f_{020}+\frac{1}{\mu_2^2}\left(\sigma_{v_2}^2+\sigma_{x_2}^2\right)f_{002}\right.$$

$$+\frac{2}{\mu_1}\sigma_{(y,x_1)}f_{110}+\frac{2}{\mu_2}\sigma_{(y,x_2)}f_{101}+\frac{2}{\mu_1\mu_2}\sigma_{(x_1,x_2)}f_{011}$$

$$+\frac{2}{2!m}\left(\sigma_u^2+\sigma_y^2\right)f_{200}+\frac{1}{\mu_1^2}\left(\sigma_{v_1}^2+\sigma_{x_1}^2\right)f_{020}+\frac{1}{\mu_2^2}\left(\sigma_{v_2}^2+\sigma_{x_2}^2\right)f_{002}$$

$$\left.+\frac{2}{\mu_1}\sigma_{(y,x_1)}f_{110}+\frac{2}{\mu_2}\sigma_{(y,x_2)}f_{101}+\frac{2}{\mu_1\mu_2}\sigma_{(x_1,x_2)}f_{011}\right]$$

$$\text{Bias}\left(\bar{y}_{aj}\right)=0 \qquad (3.19)$$

Squaring both side of (3.18) and taking expectation, the mean square error of $\bar{y}_{aj}$, upto terms of $O\left(\frac{1}{n}\right)$, is given by

$$MSE\left(\bar{y}_{aj}\right)=E\left[\frac{2}{(2m)^{1/2}}\left\{\left(W_u+W_y\right)f_1+\frac{1}{\mu_1}\left(W_{v_1}+W_{x_1}\right)f_2+\frac{1}{\mu_2}\left(W_{v_2}+W_{x_2}\right)f_3\right\}\right.$$

$$-\frac{1}{2}\frac{1}{(m)^{1/2}}\left\{\left(w_{u^{(1)}}+w_{y^{(1)}}\right)f_1+\frac{1}{\mu_1}\left(W_{v_1^{(1)}}+W_{x_1^{(1)}}\right)f_2+\frac{1}{\mu_2}\left(W_{v_2^{(1)}}+W_{x_2^{(1)}}\right)f_3\right\}$$

$$\left.-\frac{1}{2}\frac{1}{(m)^{1/2}}\left\{\left(w_{u^{(2)}}+w_{y^{(2)}}\right)f_1+\frac{1}{\mu_1}\left(W_{v_1^{(2)}}+W_{x_1^{(2)}}\right)f_2+\frac{1}{\mu_2}\left(W_{v_2^{(2)}}+W_{x_2^{(2)}}\right)f_3\right\}\right]^2$$

$$=\left[\frac{4}{2m}\left\{\left(\sigma_u^2+\sigma_y^2\right)\left(f_1\right)^2+\frac{1}{\mu_1^2}\left(\sigma_{v_1}^2+\sigma_{x_1}^2\right)\left(f_2\right)^2+\frac{1}{\mu_2^2}\left(\sigma_{v_2}^2+\sigma_{x_2}^2\right)\left(f_3\right)^2\right.\right.$$

$$\left.+\frac{2}{\mu_1}\sigma_{(y,x_1)}f_1f_2+\frac{2}{\mu_2}\sigma_{(y,x_2)}f_3f_1+\frac{2}{\mu_1\mu_2}\sigma_{(x_1,x_2)}f_2f_3\right\}$$

$$+\frac{1}{4m}\left\{\left(\sigma_u^2+\sigma_y^2\right)\left(f_1\right)^2+\frac{1}{\mu_1^2}\left(\sigma_{v_1}^2+\sigma_{x_1}^2\right)\left(f_2\right)^2+\frac{1}{\mu_2^2}\left(\sigma_{v_2}^2+\sigma_{x_2}^2\right)\left(f_3\right)^2\right.$$

$$\left.+\frac{2}{\mu_1}\sigma_{(y,x_1)}f_1f_2+\frac{2}{\mu_2}\sigma_{(y,x_2)}+f_3f_1+\frac{2}{\mu_1\mu_2}\sigma_{(x_1,x_2)}f_2f_3\right\}$$

$$+\frac{1}{4m}\left\{\left(\sigma_u^2+\sigma_y^2\right)\left(f_1\right)^2+\frac{1}{\mu_1^2}\left(\sigma_{v_1}^2+\sigma_{x_1}^2\right)\left(f_2\right)^2+\frac{1}{\mu_2^2}\left(\sigma_{v_2}^2+\sigma_{x_2}^2\right)\left(f_3\right)^2\right.$$

$$\left.+\frac{2}{\mu_1}\sigma_{(y,x_1)}f_1f_2+\frac{2}{\mu_2}\sigma_{(y,x_2)}f_3f_1+\frac{2}{\mu_1\mu_2}\sigma_{(x_1,x_2)}f_2f_3\right\}$$

$$-\frac{4}{\sqrt{2}2(2m)^{\frac{1}{2}}m^{\frac{1}{2}}}\left\{\left(\sigma_u^2+\sigma_y^2\right)\left(f_1\right)^2+\frac{1}{\mu_1^2}\left(\sigma_{v_1}^2+\sigma_{x_1}^2\right)\left(f_2\right)^2+\frac{1}{\mu_2^2}\left(\sigma_{v_2}^2+\sigma_{x_2}^2\right)\left(f_3\right)^2\right.$$

$$\left.+\frac{2}{\mu_1}\sigma_{(y,x_1)}f_1f_2+\frac{2}{\mu_2}\sigma_{(y,x_2)}f_3f_1+\frac{2}{\mu_1\mu_2}\sigma_{(x_1,x_2)}f_2f_3\right\}$$

$$-\frac{4}{\sqrt{2}2(2m)^{\frac{1}{2}}m^{\frac{1}{2}}}\left\{\left(\sigma_u^2+\sigma_y^2\right)\left(f_1\right)^2+\frac{1}{\mu_1^2}\left(\sigma_{v_1}^2+\sigma_{x_1}^2\right)\left(f_2\right)^2+\frac{1}{\mu_2^2}\left(\sigma_{v_2}^2+\sigma_{x_2}^2\right)\left(f_3\right)^2\right.$$

$$\left.\left.+\frac{2}{\mu_1}\sigma_{(y,x_1)}f_1f_2+\frac{2}{\mu_2}\sigma_{(y,x_2)}f_3f_1+\frac{2}{\mu_1\mu_2}\sigma_{(x_1,x_2)}f_2f_3\right\}\right]$$

$$=\frac{1}{2m}\left[\left(\sigma_u^2+\sigma_y^2\right)\left(f_1\right)^2+\frac{1}{\mu_1^2}\left(\sigma_{v_1}^2+\sigma_{x_1}^2\right)\left(f_2\right)^2+\frac{1}{\mu_2^2}\left(\sigma_{v_2}^2+\sigma_{x_2}^2\right)\left(f_3\right)^2\right]$$

$$+\frac{2}{2m}\left[\frac{1}{\mu_1}\sigma_{(y,x_1)}f_1f_2+\frac{1}{\mu_1\mu_2}\sigma_{(x_1,x_2)}+f_2f_3+\frac{1}{\mu_2}f_3f_1\sigma_{(y,x_2)}\right]$$

$$MSE\left(\bar{y}_{aj}\right)=\frac{1}{2m}\left[\sigma_y^2\left(f_1\right)^2+\frac{1}{\mu_1^2}\sigma_{x_1}^2\left(f_2\right)^2+\frac{1}{\mu_2^2}\sigma_{x_2}^2\left(f_3\right)^2\right.$$

$$\left.+2\left\{\frac{1}{\mu_1}\sigma_{(y,x_1)}f_1f_2+\frac{1}{\mu_1\mu_2}\sigma_{(x_1,x_2)}f_2f_3+\frac{1}{\mu_2}\sigma_{(y,x_2)}f_3f_1\right\}\right]$$

$$+\frac{1}{2m}\left[\sigma_u^2\left(f_1\right)^2+\frac{1}{\mu_1^2}\sigma_{v_1}^2\left(f_2\right)^2+\frac{1}{\mu_2^2}\sigma_{v_2}^2\left(f_3\right)^2\right] \quad (3.20)$$

IV.CONCLUDING REMARKS

1. We can easily see that bias and mean square error of $\bar{y}_a$ are

$$Bias\left(\overline{y}_a\right)=\frac{1}{2n}\left[\sigma_y^2 f_{200}+\frac{1}{\mu_1^2}\sigma_{x_1}^2 f_{020}+\frac{1}{\mu_2^2}\sigma_{x_2}^2 f_{002}+2\left\{\frac{\sigma_{(y,x_1)}}{\mu_1}f_{100}+\frac{\sigma_{(x_1,x_2)}f_{011}}{\mu_1\mu_2}+\frac{\sigma_{(y,x_2)}}{\mu_2}f_{101}\right\}\right]$$
$$+\frac{1}{2n}\left[\sigma_u^2+\frac{\sigma_{v_1}^2}{\mu_1^2}f_{020}+\frac{\sigma_{v_2}^2}{\mu_2^2}f_{002}\right] \quad (4.1)$$

and mean square error be

$$MSE\left(\overline{y}_a\right)=\frac{1}{n}\left[\sigma_y^2\left(f_1\right)^2+\frac{\sigma_{x_1}^2}{\mu_1^2}\left(f_2\right)^2+\frac{\sigma_{x_2}^2}{\mu_2^2}\left(f_3\right)^2+2\left\{\frac{\sigma_{(y,x_1)}}{\mu_1}f_1f_2+\frac{\sigma_{(x_1,x_2)}}{\mu_1\mu_2}f_2f_3+\frac{\sigma_{(y,x_2)}}{\mu_2}f_3f_1\right\}\right]$$
$$+\frac{1}{n}\left[\sigma_u^2\left(f_1\right)^2+\frac{\sigma_{v_1}^2}{\mu_1^2}\left(f_2\right)^2+\frac{\sigma_{v_2}^2}{\mu_2^2}\left(f_3\right)^2\right] \quad (4.2)$$

Further, from (3.19) and (3.20), the bias and mean square error of jack-knifed estimator

$$Bias\left(\overline{y}_{aj}\right)=0 \quad (4.3)$$

$$MSE\left(\overline{y}_{aj}\right)=\frac{1}{2m}\left[\sigma_y^2\left(f_1\right)^2+\frac{1}{\mu_1^2}\sigma_{x_1}^2\left(f_2\right)^2+\frac{1}{\mu_2^2}\sigma_{x_2}^2\left(f_3\right)^2\right.$$
$$\left.+2\left\{\frac{1}{\mu_1}\sigma_{(y,x_1)}f_1f_2+\frac{1}{\mu_1\mu_2}\sigma_{(x_1,x_2)}f_2f_3+\frac{1}{\mu_2}\sigma_{(y,x_2)}f_3f_1\right\}\right]$$
$$+\frac{1}{2m}\left[\sigma_u^2\left(f_1\right)^2+\frac{1}{\mu_1^2}\sigma_{v_1}^2\left(f_2\right)^2+\frac{1}{\mu_2^2}\sigma_{v_2}^2\left(f_3\right)^2\right] \quad (4.4)$$

From (4.2) and (4.4), we see that both estimators $\overline{y}_a$ and $\overline{y}_{aj}$ have the same mean square error but from (4.1) bias of $\overline{y}_a$ is not zero whereas from (4.3) bias of the jack-knifed estimator $\overline{y}_{aj}$ is zero and both the estimators $\overline{y}_a$ and $\overline{y}_{aj}$ having the same mean square error, the jack-knifed estimator $\overline{y}_{aj}$ may be preferred to the estimator $\overline{y}_a$ in the presence of measurement errors also.

Therefore, the proposed unbiased jack-knifed estimator should be preferred than the conventional estimators under measurement error as they help in removing the bias while still preserving the efficiency. Many important estimators like that of proposed by Abu-dayyeh et al. (2003) given by

$$\overline{y}_\alpha=\overline{y}\left(\frac{\overline{x}_1}{\mu_1}\right)^{\alpha_1}\left(\frac{\overline{x}_2}{\mu_2}\right)^{\alpha_2} \quad (4.5)$$

are special cases of the present study when we assume the measurement errors to be absent. The conventional results can be obtained as a special case by setting the measurement error variances to be zero.

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